STABILITY OF VERTICAL BACK AND BATTERED BACK DAM SECTIONS

BY

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AN INVESTIGATION OF THE RELATIVE STABILITY

OF

VERTICAL BACK AND BATTERED BACK DAM SECTIONS

A THESIS

PRESENTED BY

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....

AND

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AN INVESTIGATION OF THE COMPARATIVE STABILITY OF STRAIGHT BACK AND BATTERED BACK DAM SECTIONS.

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Numerous recent failures of dams, built in accordance with the criterions for the practical section so long in use, have shown forcibly the necessity for the consideration of additional factors in the design of dams subjected to high heads of water.

These considerations suggest the adoption of a new section - viz., one in which the head of water itself is used to lend additional stability, by taking advantage of its vertical component on a battered back.

It has been recently emphasized, that the most important of these new factors is the effect of uplift due to the seepage of water under the foundation and into the body of the dam itself. Various assumptions have been made as to the amount of this uplift.

It is only reasonable to assume, that at least one-third of the foundation area is in bearing, and that this area is impervious. In a well laid foundation, the amount of the seepage water which finds an outlet at the toe is exceedingly small, and

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is under no head, while that which is retained is under the full static head at the heel and, though it may be distributed through very small pores, the summation of the whole becomes a factor to be reckoned with.

The existence of the upward pressure under a dam depends on the relative perviousness of the material under the dam, rather than on the absolute quantity of percolation.

In the extreme case, the pressure varies from the total static head at the heel to zero at the toe. It is certain that the water has no access to that part of the foundation which is in bearing; hence, if this area is assumed to be one-third of the whole, then two-thirds of the foundation (or of any joint), is subject to uplift. This is the assumption used in the investigation which follows.

Another factor which has been suggested is that due to the ice pressure. What this pressure will be depends on the thickness of the ice and other local conditions at the individual site. The period of heaviest ice pressure usually occurs at the time when the water is low, and the pressure will be applied at a point considerably below high water, where the dam

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will be strong enough to resist the additional pressure. The uplift of water is of greater importance than ice pressure. The ice is in sight and can be cut from the face of the dam, should the necessity arise, thus relieving the pressure. Different problems involve such varied circumstances, that no general rule can be applied, and no attempt will be made to incorporate this factor in the discussion to follow.

Consideration of the nature of the foundation has been neglected in this comparison, for the reason that each site resolves itself into a particular problem, and involves considerations apart from the subject of this thesis. For instance: In a dam resting on yielding soil, the uplift will be of greater moment than it is with one resting on a solid foundation, because concentration of pressure on a yielding base is more likely to cause failure, and the softer soil permits freer sliding.

Under drainage is ineffectual, for the reason that the entire dam would have to be honeycombed, thereby wasting too much water to be practical as a system. The same outlay made for more masonry would be of more lasting good.

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Analysis of Section I
Considered Without Uplift.

This section was drawn up in accordance with the accepted rules for the standard type of gravity dam, as shown on Plate VIII, but all consideration of the effect of uplift was neglected for the sake of comparison with the section of Plate I, which particularly involves this figure.

Theoretically, the thickness of a dam runs to zero at the top, but, because of the advantages of the wider top, this one is chosen as 10' - 0" thick at the flood water level (Elev.100.0). As usually designed, the top of the dam is above the high water, a distance of 5 to lo-feet,- in this case 10' - 0".

The trial sections were chosen of such proportions that the following limiting functions were complied with:

1. The lines of pressure both for the reservoir full and empty, must not pass outside the middle third of any horizontal joint.

2. Maximum working pressure on any horizontal joint must never exceed certain prescribed limits, either in the masonry itself, or in the foundation. (See data of Plates VIII and IX).

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2. Maximum working pressure on any horizontal joint must never exceed certain prescribed limits, either in the masonry itself, or in the foundation. (See data of Plates VIII and IX).

3. The coefficient of friction in any plane horizontal joint, or between the dam and its foundation, must be less than the tangent of the angle which the resultant makes with the vertical.

This last feature will not be treated here, owing to the fact that the quality of masonry, and the nature of the foundation are of such extensive detail that they become a subject by themselves. In passing it may be well to add, that in regard to construction, the tendency should be to bring the quality up to the requirements of good design, rather than make allowances, and conform the design to work of poor quality.

Compliance with the limiting functions previously enumerated, and observance of certain other precautions, which may be demanded by the special case, will insure safety against the following methods of failure:

1. By overturning about the front of any horizontal section.

2. By crushing the masonry as -

(a) At the downstream edge of the horizontal section, with the reservoir full.

(b) At the upsteam edge, with the reservoir empty.

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(a) At the downstream edge of the horizontal section, with the reservoir full.

(b) At the upstream edge, with the reservoir empty.

3. By sliding along a horizontal joint, or shearing along any section.

For an economical section the proportion of base to height should be made as small as possible within the range of safety. This ratio is usually about 7/10 for the type of Plate VIII.

Having determined the mass of an assumed section, and computed the concentrated pressure of the water, together with its point of application, the direction and magnitude of the resultant pressure were graphically determined for reservoir, both full and empty. The base of each section may be considered as a joint.

It was found convenient to use a slight batter on the back of the dam. Its effect toward increased stability, by the addition of a vertical component of water pressure, is very slight, as may be seen from the small displacement between the solid pressure line and the dotted one (connecting points of intersection between resultants and their respective joints), in the determination of which the vertical component of static pressure was neglected.

The final computations for this simple section will be found in Table I, Page VII. These

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Straight-Back Section (I) Without Uplift.

Sect. No.	Depth	Base	Wts. in Tons: By Sec.	Wts. in Tons: Comb.	C. of G.: Dist	C. of G.: Moved	Water: H. Comp	Water: V. Comp	Water: Uplift	Effect. Vert.	Unit Shear	Ratio H/V
1	10'-0"	10'-0"	7.50		ft.	ft.		½ Batter starts at base Sect.		7.50	lbs. per sq. ft.	
2	5'-0"	do	3.75	11.25			.39		Trial made at foundation only.	11.25	78.1	.035
3	5'-0"	10'-3"	3.80	15.05	10.00	3.38	1.56			15.04	305	.104
4	10'-0"	12'-0"	8.35	23.40	14.33	5.11	6.25			23.39	1042	.266
5	10'-0"	16'-6"	10.69	34.09	19.50	6.11	14.06	.054		34.13	1702	.411
6	10'-0"	23'-0"	14.81	48.90	23.75	9.21	25.00	.131		49.04	21.72	.508
7	12'-0"	32'-3"	24.86	73.76	26.00	8.76	42.27	.257		73.88	26.22	.570
8	12'-0"	42'-0"	35.00	108.76	29.40	9.07	63.00	.402		107.31	3000	.587
9	12'-0"	52'-0"	41.89	150.65	32.67	9.21	89.75	.584		149.49	3450	.598
10	12'-0"	62'-0"	51.30	201.95	36.33	9.32	121.00	.797		201.15	3900	.608
11	12'-0"	72'-0"	69.30	271.25	39.50	9.45	156.25	1.042	75.15	220.87	4340	.578
Totals	110'-0"		271.25	All weights in tons except as noted.								

Table I.
Thesis.
B. R.

Sec. No.	Max Heel & Toe Press. Empty Heel	Max Heel & Toe Press. Empty Toe	Max Heel & Toe Press. Full Heel	Max Heel & Toe Press. Full Toe	Eccentr'ty (ft.) Empty	Eccentr'ty (ft.) Full	Factor of Safety	Vertical Water. Comp. @ Base	Vertical Water. End Av.	Vertical Water. Hor. Proj.
1							∞	Pounds per foot	½ Batter starts at base Sec.	See Table I for product
2	—	—	1.09	1.15	0.0	.04	86.3			
3	.75	2.19	1.34	1.66	.08	.17	15.3			
4	1.14	2.76	1.15	3.03	.83	.87	3.82	104		
5	1.18	2.94	.44	4.47	1.75	2.12	2.45	156	130	0.83
6	.46	3.79	.19	5.23	3.00	3.17	2.18	208	156	1.67
7	.08	4.50	.53	5.69	5.00	4.08	2.14	281	193	2.67
8	.03	5.05	.73	6.11	6.92	4.75	2.15	333	219	3.67
9	.05	5.67	1.14	6.69	8.50	5.33	2.24	396	250	4.67
10	.17	6.29	1.00	7.34	9.83	6.38	2.33	458	281	5.67
11	.33	7.15	1.71	8.29	10.92	6.83	2.47	521	313	6.67
All pressures in tons except as noted.										

Table Ia
Thesis

B. B.

results are self explanatory, with the possible exception of those for center of gravity distance. By this is meant the distance (in feet) from the center of gravity of a combined mass, or group of sections, to that of the section under consideration. And the distance "moved" is that determined by the equation.

$$x = \frac{m_s d}{m_s + m_l}$$

m_s is mass of new section under consideration.

m_l is mass of combined sections above whose c. of g. has been located.

d is c. of g. distance previously explained. A point along the line connecting the center of gravity of m_l to that of m_s at a distance X from the first locates the new c. of g.

The vertical component of the water pressure at a point is

$$v = wh \tan \theta$$

w = weight of cubic feet of water.

h = head on base of section.

θ = angle of the battered surface with the vertical.

To find the pressure over a certain section of back, the components of the limiting points were averaged and then multiplied by the horizontal projection of the batter. The product gives the resultant, which acts through the center of gravity of the limit-

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$$Y = \frac{m_1 d}{m_1 + m_2}$$

m_1 is mass of new section under consideration.

m_2 is mass of combined sections above whose c. of g. has been located.

d is c. of g. distance previously explained. A point along the line connecting the center of gravity of m_2 to that of m_1 at a distance Y from the first locates the new c. of g.

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In Table Ia, under "Components at Base," will be found the results just mentioned as "Limiting Values." In the next column are "End Averages," and the product of these last, with the corresponding "Horizontal Projections," gives the "Vertical Components" found in Table I.

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Analysis of Section II;

Designed for Uplift.

This section was designed in the same wayas was Section I, except that the effect of uplift was taken into consideration for each section. Since the amount of this uplift is taken as 2/3 the static head, as previously explained,

$$U = \frac{2}{3}\,\frac{hwb}{2} = \frac{hwb}{3}.$$

This force is applied vertically under the middle third point adjacent to the heel, and increases the overturning moment. For if moments are taken about the toe of a section, the components for weight of masonry and vertical water pressure tend to revolve in one direction, while those of uplight and horizontal water pressure act in the opposite direction.

To offset the effect of uplift additional masonry is required, and the ratio of base to height is increased over that for the same sections of Plate VIII.

Here again it was found convenient to batter the back. 1 : 6 was used,- just double that for the profile of no uplift.

In the final construction the uplift being

opposed to the larger vertical forces, shortens their component, and increases the angle of the resultant with the vertical proportionally.

The smallest factor of safety in Section I was 2.14, whereas it drops to 1.8 at one point in Section II, even though sufficient cross-sectional area is provided to cause the pressure lines to fall within the middle third.

The trial made at the foundation of Section I for the effect of uplight, shows to what extent the pressure line falls outside the lower middle third in a standard design, satisfying all but this very important condition. It is evident that a large overturning moment is added, and tension is induced in the masonry at the heel.

Efforts to prevent water from entering masonry walls are strenuous enough to convince the designer that water will certainly penetrate a dam, and require provision for its maximum effect.

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Straight-Back Section (II) With Uplift.												
Sec. No.	Depth	Base	Wts. in Tons By Sec	Comb	C. of G. Dist.	Moved	Water H. Comp	V. Comp	Uplift	Effect Vert.	Unit Shear	Ratio H/V
1	10'-0"	10'-0"	7.50		ft	ft.		$\frac{1}{6}$ Batter starts at base Sec. 4		7.50	lbs per sq ft	
2	5'-0"	do	3.75	11.25			.99		.52	10.73	78.1	.037
3	5'-0"	11'-0"	3.94	15.19	10.08	2.60	1.96		1.4	14.04	284.2	.111
4	10'-0"	15'-0"	9.75	24.94	15.33	6.02	6.25		3.13	21.81	834.	.287
5	10'-0"	21'-8"	13.75	38.69	19.58	7.02	14.06	.217	6.87	32.03	1297	.438
6	10'-0"	29'-4"	19.13	57.82	22.58	7.58	25.00	.511	12.22	46.10	1707	.542
7	12'-0"	39'-4"	30.90	88.72	26.42	9.17	42.27	1.000	21.30	68.40	2150	.618
8	12'-0"	50'-4"	40.35	129.07	29.50	9.20	63.00	1.603	33.55	97.10	2500	.648
9	12'-0"	61'-4"	50.25	179.32	32.83	9.08	89.75	2.332	48.53	133.10	2930	.674
10	12'-0"	73'-4"	60.60	239.92	36.00	9.08	121.00	3.187	67.22	175.87	3300	.688
11	12'-0"	85'-4"	71.40	311.32	39.67	9.08	156.25	4.165	88.89	226.57	3670	.689
Totals	110'-0"		311.32	All weights in tons except as noted.								

Table II.
Thesis

Sec. No.	Max. Heel & Toe Press. Empty Heel	Empty Toe	Full Heel	Full Toe	Eccentr'ty Empty (ft.)	Eccentr'ty Full (ft.)	Factor of Safety	Vertical Water Comp. @ Base	Vertical Water End Av.	Vertical Water Hor Proj.
								Pounds per Foot	½ Batter starts at base Sec. 4	See Table II for product
1		.75								
2	—	1.13	.90	1.23	—	.25	2.36			
3	1.06	1.70	1.17	1.41	.42	.17	2.28			
4	.22	3.11	1.59	1.78	2.17	.13	2.30	208		
5	.43	3.13	1.38	2.10	2.75	1.71	2.28	313	260	1.67
6	.56	3.38	.38	3.61	3.50	3.42	1.99	417	313	3.33
7	.63	3.70	.13	4.63	4.75	5.30	1.80	542	375	5.33
8	.68	4.46	.21	5.31	6.17	6.50	1.90	667	438	7.33
9	.70	5.15	.0	6.15	7.75	8.33	2.04	792	500	9.33
10	.78	5.76	.21	6.84	9.33	9.50	2.11	917	563	11.33
11	.93	6.36	.34	7.47	10.58	10.70	2.20	1042	630	13.33

Table title: Straight Back Section (II) With Uplift.

All pressures in tons except as noted.

Table IIa.
Thesis.

JB RE

Analysis of the Design of Section IV
45° Back.

Inasmuch as the batter of 12:12 resolves the forces into vertical and horizontal forces that are equal, this type of section was the first to be adopted in considering the possibility of using an inclined back to overcome the effect of the uplift due to the seepage water.

According to all accepted designs, the top of the dam should be at least equal to 1/10 of the height of water at flood level, and should project above high water the same amount. In order to do this, and use a 45-deg. back, it is necessary to make the base of the dam 119', which is much in excess of the bade required for a straight back dam of the same height. This condition suggested the possibility of using a section that is composed of a hollow concrete shell filled with rough fitted dry masonry, under the assumption that the masonry would take the direct pressure, and relieve the face and diaphragms of any stress due to bending, but will resist the shear by friction, only. In carrying out this idea, the

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proportions used were as follows: A four foot concrete face, waterproofed, supported every 20' in both the vertical and the horizontal directions by four foot concrete diaphragms, to be made of concrete weighing 150-lbs. per cubic foot, the cavities formed by these diaphragms to be filled with limestone masonry, fitted roughly but laid up dry, and weighing 130-lbs. to the cubic foot.

The design of the dam was made by purely graphical methods, and in outline was as follows:

The dam was divided into eleven unequal parts, the top two being concrete, and every alternate one from there to the foundation being concrete also.

For Section II in this dam, the calculations were as follows: The section is in the form of a trapezoid, ten feet wide on top and fifteen feet wide at the bottom. The volume of this trapezoid would be

$$\frac{10+15}{2} \times 5 = 62.5 \text{ cubic feet.}$$

Due to the fact that the area of concrete available to take the shear, an additional factor of safety was introduced here by using the weight of concrete as 140-lbs.

The total weight of the section was

$$62.5 \times 140 = 8750.$$

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top to a maximum of 312.5-lbs. per square foot at the bottom. This gives a total pressure on the back of the section of -

$$\frac{312.5}{2} \times 6.2 = 971.54\text{-lbs.}$$

where 6.2' is the inclined length of the back of the dam. The water can be considered as acting perpendicular to the face of the dam, through the center of gravity of the polygon of forces. In this case this will be at a height of one-third of the head of water, or 1.66' above the base of the section.

The center of gravity of the section is at a distance of 2.5' above the base, and 6.2' from the lower downstream face. The coordinates of the mass above are 5%; the distance between the two centers is 7.8'.

If w is the weight of all the mass above the section, and w' is the mass of all sections above, including the section considered, d is the distance between the two centers, and x is the distance from the center of gravity of the section to the center of gravity of the total mass, measured along the line from the center of the section and of the total section above, then x equals -

$$x = \frac{14000 \times 7.8}{22750} = \frac{w\ d}{w'} = 4.8'$$

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$$\frac{312.5}{2} \times 6.2 = 971.54\text{-lbs.}$$

where 6.2' is the inclined length of the back of the dam. The water can be considered as acting perpendicular to the face of the dam, through the center of gravity of the polygon of forces. In this case this will be at a height of one-third of the head of water, or 1.66' above the base of the section.

The center of gravity of the section is at a distance of 2.5' above the base, and 6.2' from the lower downstream face. The coordinates of the mass above are 5'; the distance between the two centers is 7.8'.

If w is the weight of all the mass above the section, and W' is the mass of all sections above, including the section considered, d is the distance between the two centers, and x is the distance from the center of gravity of the section to the center of gravity of the total mass, measured along the line from the center of the section and of the total section above, then x equals -

$$x = \frac{1500 \times 7.8}{2745} = \frac{wd}{W'} = 4.8'$$

With the intersection of the gravity line and the line of water pressure as the origin, lay off the polygon of forces and get the resultant. The resultant cuts the base of the section at a point 2.7' inside of the middle third, and is equal to 15,200-lbs. at high water. At low water the resultant cuts the base at a point .8' inside of the middle third.

For Section III, which is part concrete and part masonry, the procedure is the same, except that the center of gravity of the section has to be determined by taking moments of the three sections (two concrete and one stone), in both the horizontal and the vertical planes, about the lower downstream corner of the section. Then solve for the coordinates of the c. of g. of the combined section.

The total shear at the bottom of the section is equal to the horizontal component of the water pressure on the back of the dam, and, inasmuch as the area of the concrete at the bottom of the dam is the same as it is at any section, it is only necessary to consider it for the bottom section. The total water pressure is equal to 219 tons. The horizontal component of this is 155 tons.

With the intersection of the gravity line and the line of water pressure as the origin, lay off the polygon of forces and get the resultant. The resultant cuts the base of the section at a point 2.7' inside of the middle third, and is equal to 15,700-lbs. at high water. At low water the resultant cuts the base at a point .8' inside of the middle third.

For Section III, which is part concrete and part masonry, the procedure is the same, except that the center of gravity of the section has to be determined by taking moments of the three sections (two concrete and one stone), in both the horizontal and the vertical planes, about the lower lowstream corner of the section. Then solve for the coordinates of the c. of g. of the combined section.

The total shear at the bottom of the section is equal to the horizontal component of the water pressure on the back of the dam, and, inasmuch as the area of the concrete at the bottom of the dam is the same as it is at any section, it is only necessary to consider it for the bottom section. The total water pressure is equal to 219 tons. The horizontal component of this is 15[illegible] tons.

The total resisting area of the concrete is 9.6' square feet. Considering the coefficient of friction of the stone as .75, the resistance to sliding offered by the stone will be -

419 x .75 = 314 tons,

which shows that the section is safe in shear.

Due to the pervious nature of the inside of the dam, all the water that can get through the back can drain through the interior without resistance, and will be taken care of by drains at the face. The up-lift on the relatively small area in the back will not be great. It is, therefore, not necessary to take into account the uplift in the body of the dam, but only that at the foundation. The total upthrust is 112 tons. This force, combined with the forces due to the mass, and the water behind the dam, throw the resultant outside of the middle third. The section, therefore, is unstable.

The total resisting area of the concrete is 9.5' square feet. Considering the coefficient of friction of the stone as .75, the resistance to sliding offered by the stone will be -

$$419 \times .75 = 314 \text{ tons.}$$

which shows that the section is safe in shear.

Due to the pervious nature of the inside of the dam, all the water that can get through the back can drain through the interior without resistance, and will be taken care of by drains at the face. The uplift on the relatively small area in the back will not be great. It is, therefore, not necessary to take into account the uplift in the body of the dam, but only that at the foundation. The total upthrust is 112 tons. This force, combined with the forces due to the mass, and the water behind the dam, throw the resultant outside of the middle third. The section, therefore, is unstable.

Uniform. Battered. Back Section (45°)

Resultant.	Coordinates of. C of G		Water			Total. Mass.	Combined Mass.	Concrete.			Stone			Width of Base	Height of Section	Sec.
	y	x	Pt. of Applicat'n	Total.	Head.			Mass	Back Width	Face Width.	Mass	Bottom Width	Top Width			
7.	5.0'	5.0'	0	0	0	7.	7.	7.	—	—	0	0	0	10'0"	10'	1
13.0	2.5'	6.5'	1.66'	562.3	5'	11.37	4.37	4.37	—	—	0	0	0	15'0"	5	2
42.	6.59'	12.49'	6.66'	8.76	20'	34.61	23.24	10.1	5.'7"	4.'0"	13.16	21.'7"	5.'5"	31'0"	15	3
53.	1.8	17	8.'	12.82	24'	43.94	9.33	9.33	—	—	0	0	0	35.'7"	4	4
113.	7.23'	22.06'	13.33.'	35.22	40'	90.17	46.73	10.75	5.'7"	4.'0"	35.98	43.'2"	26'0"	52.'8"	16	5
141.5	1.7	27.8'	14.66'	42.57	44'	107.14	16.47	16.47	—	—	0	0	0	57'0"	4	6
238.	7.4	33.78'	20'	78.97	60'	176.03	68.38	10.75	5.'7"	4.'00"	58.13	64.'5"	64.'0"	74'0"	16	7
285.	1.8'	38.4'	21.33'	89.75	64	198.89	228.6	22.86	—	0	0	0	0	78.'4"	4	8
404.	7.65	43.7'	26.66'	140.15	80'	290.35	91.45	10.75	5.'7"	4.'0"	80.71	86.'5"	68.'8"	96'0"	16	9
440.	1.8'	49.4'	28'	154.50	84'	317.84	27.49	27.49	—	—	0	0	0	100.'3"	4	10
600.	7.8'	54.4'	33.33'	218.90	100'	430.16	112.81	10.75	5.'7"	4.'0"	102.1	107.'4"	90.'8"	117.'0"	16	11

Note: All Masses given in tons.

Table. IV.
Thesis
R.B.C.

Uniform - Battered Back Section (45°)

Max. Toe and Heel. Pressures				Factor of Safety.	Ratio of Hor. to Ver. Forces (μ.)	Unit Shear.	Total. Vertical Force	Water. compont.		Eccentricities		Sec.
Empty.		Full										
Heel.	Toe	Heel	Toe.					Vertical	Horigont.	Empty.	Full	
.7	.7	.7	.7	∞	0	0	14000	0	0	0	0	1
.192	1.32	.435	1.335	96.9	.0352	.05325	11.769	.398	.399	2'0"	1'3"	2
.082	2.157	.412	2.377	17.25	.0955	.213	37.978	3.311	3.311	3'9"	3'8"	3
.086	2.296	.415	2.615	14.69	.132	.267	48.692	4.751	4.751	4'2"	4.2"	4
.045	3.395	.385	4.005	13.12	.14	.481	103.380	12.703	12.703	6'10"	6'8"	5
.115	3.655	.652	4.517	12.42	.141	.537	122.459	15.312	15312	7'0"	6'10"	6
.22	4.54	.942	5.667	11.89	.1.61	.763	204.28	28.24	28.24	9'0"	8'9"	7
.292	4.782	.970	6.450	10.45	.162	.82	230.966	32.071	32.071	9'9"	9'6"	8
.385	5.675	1.20	7.525	9.95	.168	1.04	340.314	49.963	49.963	11'7"	11'2"	9
.475	5.785	1.235	7.875	9.9	.176	1.11	372.954	55.057	55.057	12'4"	11'10"	10
.437	6.987	1.915	9.315	11.6	.182	1.36	488.658	57.97	57.97	15'0"	14'2"	11

Table IV a.
Thesis.
FB. JB.

Analysis of Section III;
Back Batter Adjusted to Take the Uplift.

Several methods were tried to bring the resultant inside of the middle third, without losing the idea of having all the batter on one side of the dam. Increasing the base increases the uplift more rapidly than it does the mass, so that it cannot help. The idea had to be finally abandoned in part, and some of the batter put on the downstream side. After several trials, it was found that by changing the face batter uniformly from zero at the top down to a maximum of 6" in 12", that the uplift could be taken care of, and that the mass of the dam could be reduced by reducing the thickness of the upper part, and confining the maximum back batter of 12" in 12" to the bottom 56' only. This change in the shape gives a section that weighs only 376 tons, in place of the 431 tons of Section IV.

In order to bring the center of gravity down father into the dam, and bring it over nearer the center, it was found more convenient to vary the sections used, and two sections of concrete, one 8'

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In order to bring the center of gravity down father into the dam, and bring it over nearer the center, it was found more convenient to vary the sections used, and two sections of concrete, one 5'

and the other 12', were used in place of the three sections of 4' each, used in dam IV. With the additional exception of the method of obtaining the resultant water pressures, the design of the section was identical in every respect.

Due to the variation in the batter on the back, it was necessary to lay out the trapezoid of forces on the back of every section, and then combine the resultant force with the resultant of all the forces above it.

This method gives a resultant slightly less than in the other dam, but it is applied at a point lower down on the back.

These resultant pressures do not act strictly perpendicular to the section to which they apply, but the variation is so slight, that the error introduced is very small and the values in Table IIIa are obtained on the assumption that they do not act perpendicularly.

The effect of the seepage water in the body of the dam is disposed of in the same manner that it was in Section IV. The effect on the base is slightly less, 110 tons, and this, combined with the other forces, brings the resultant inside of the middle Third.

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The maximum shear that will occur in this dam occurs at the bottom of Section IX, and is equal to 110 tons. The resistance offered by the friction of the masonry is .75 x 262= 196 tons, so that the shear is negligible.

The shears given in Table IIIa, and used in plotting curve 6, are based on the assumption that the shear is distributed uniformly over the total area of the section - stone and concrete alike.

COMPARISON.

In spite of the great variance between the four sections, most of the different factors show a similarity, - in some cases remarkable. In one way it is impossible to get a good comparison, due to the difference in materials used for the sections, but, for general purposes, the points of similarity and difference are -

In Sections I and II, the width of base adopted very closely agrees with the base generally used with straight back dams, viz., .7 for Section I, and .85 in Section II.

In Sections III and IV it was impossible to

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Back Batter adjusted for Uplift.

Resultant.	Coordinates of c of g.		Water.			Total Mass.	Combined Mass.	Concrete.			Stone.			Width of Base	Height	Sec
	y	x	Pt of Application	Total.	Head.			Mass.	Back width	Face width	Mass	Bottom width	Top width			
4.2	5'0"	3'0'		0	0	4.2	4.2.	4.2	—	10'	—	—	—	10'0"	6'	1
125	3'7"	7'3"		.362	4'0"	12.04	7.84.	7.84	—	10'	—	—	—	18'0"	8'	2
40.22	7'5"	10'9"		6.5	20'0"	40.14	28.4.	11.2	4'0"	4'00"	16.9	14'0"	10'0"	22'0"	16'	3
50.	1'9"	18'0"		9.0	24'0"	46.6	6.4	6.4	—	—	—	—	—	24'0"	4'	4
87.5	7'8"	19'6"		25.8	40'0"	76.3	29.7	9.18	4'1"	4'2"	20.59	23'9"	15'9"	32'0"	16'	5
100.	2'0"	17'4"		31.5	44'0"	85.7	9.3	9.4	—	—	—	—	—	34'8"	4'	6
177.5	7'3"	25'7"		60.6	60'0"	133.6	47.9	10.9	5'7"	4'2"	36.92	46'0"	25'0"	55'9"	16'	7
231.	3'6"	33'1"		87.6	68'0"	168.2	34.6	34.6	-	—	—	—	—	68'0"	8'	8
385	8'8"	46'8"		155.	88'0"	276.9	108.6	14.2	5'7"	4'7"	94.38	87'5"	57'9"	97'7"	20'	9
540.	C'4"	52'0"		203.7	100'00"	376.3	99.3	99.3.	—	—	—	—	—	115'5"	12'	10
426.	"	"	"	"	"	"	•	"	"	"	"	"	"	"	"	10a

Note:- Total uplift on foundation = 2.2 Tons
All Masses given in Tons

Table. III.

Thesis.

PB. R

[illegible]

Back-Batter adjusted for Uplift

Toe and Heel Pressures: Empty, Heel	Empty, Toe	Full, Heel	Full, Toe	Factor of Safety	Ratio of Horizontal To Vertical	Unit Shear	Total Vertical Force	Water Component: Vert.	Water Component: Hor.	Eccentricities: Empty	Eccentricities: Full	Sec.
4.2	4.2	.42	.42	∞	0	0	4.2	0	0	0	0	1
.110	1.26	.426	1.154	21.9	.21	.0143	12.29	.252	.252	2'7"	2'0"	2
1.30	2.29	1.06	2.61	10.3	.188	.292	40.25	.107	6 44	1'0"	1'9"	3
1.45	2.42	1.15	3.05	7.81	.175	.358	48.68	2.08	8.60	1'0"	1'9"	4
1.82	2.94	.36	5.24	3.60	.29	.78	84.42	8.12	24.5	.9"	4'7"	5
2.14	2.78	.27	5.72	3.77	.296	.83	97.95	12.25	29.0	9"	5'0"	6
1.15	3.63	.72	5.92	6.5	.241	.78	176 5	42 9	42.9	5'5"	7'0"	7
.65	3.81	1.24	5.91	6.5	.272	.927	231.2	63.0	63.0	6'1"	7'0"	8
1.51	4.15	2.22	6.18	6.64	.285	1.13	386.9	110	110.0	7' 7"	7'2"	9
1.85	4.66	3.85	5.83	8.17	.31	1.46	545.3	169	169.0	8'2"	3'9"	10c
		1.74	6.03	2.44	.388	1.46	435.3	59	169.0	10'0"	10'0"	10a

Table III a.
Thesis.
FB. [illegible]

bring the bases down to this low limit, The ratio being 1.08 for Section IV, and 1.06 for Section III. It was on this account, and the fact that Sections III and IV would have to be much heavier, that the particular construction used was tried, for additional factor of safety. The weight of a less dense concrete was used in Sections III and IV, but this fact did not reduce the weights of these sections down as low as the weights of I and II. This difference is very great for Sections I and IV, but slightly less for II and III. The weight per square foot of base area is least for Section III, and increases through IV and II to a maximum with I.

TOE PRESSURE; RESERVOIR FULL.

The pressures generated at the toe for any section of each of the four dams with high water, are plotted on Plate I, Page XXVIII. From these curves it can be seen that the two types - straight back and battered back, differ in that the toe pressures for the vertical back are greater at all points in the dam when considering uplift.

In the battered back dam the toe pressure becomes less in the section with uplift before the 40'

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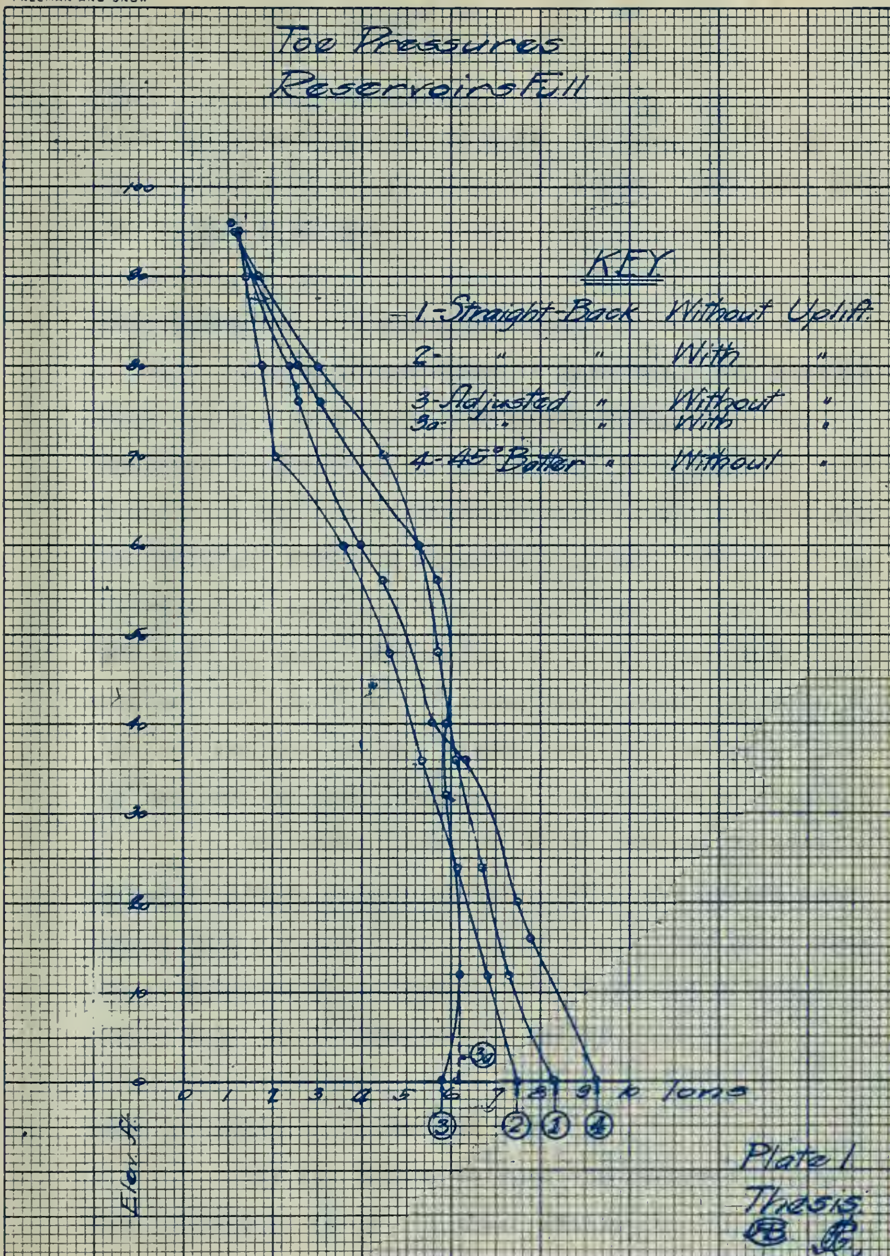
Toe Pressures
Reservoirs Full
KEY
1-Straight-Back Without Uplift
2- " " With "
3-Adjusted " Without "
3a- " " With "
4-45° Batter " Without "
100
90
80
70
60
50
40
30
20
10
0
0 1 2 3 4 5 6 7 8 9 10 Tons
3a
3 2 1 4
Elev. Ft.
Plate I
Thesis.

line the pressures are least for Section II, but at this point they all change; III becomes smallest, and IV becomes greatest.

HEEL PRESSURES: (RESERVOIR FULL):

The heel pressures at high water are plotted on Plate II, Page XXX. The curves indicate a rapid increase of pressure at the 80-foot line, falling off again to a minimum at elevation 60, at which point all sections are stressed approximately the same, with the exception of Section II, and possibly, the increase of pressure increases gradually as the foundation is approached. Section III shows the largest increase in this direction, for the reason that the center of mass is nearest the face, because the batter is on the back.

The short piece of curve numbered 3a shows a large decrease in heel pressure, when the uplift is taken into consideration. This type of dam has a wide base, because of its extensive batter, and offers a large area to the hydrostatic uplift.

TOE PRESSURES: (RESERVOIRS EMPTY).

The toe pressures with the reservoir empty are plotted on Table III, Page XXXII.

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HEEL PRESSURES: (RESERVOIR FULL):

The heel pressures at high water are plotted on Plate II, Page XXX. The curves indicate a rapid increase of pressure at the 30-foot line, falling off again to a minimum at elevation 60, at which point all sections are stressed approximately the same, with the exception of Section II, and possibly, the increase of pressure increases gradually as the foundation is approached. Section III shows the largest increase in this direction, for the reason that the center of mass is nearest the face, because the batter is on the back.

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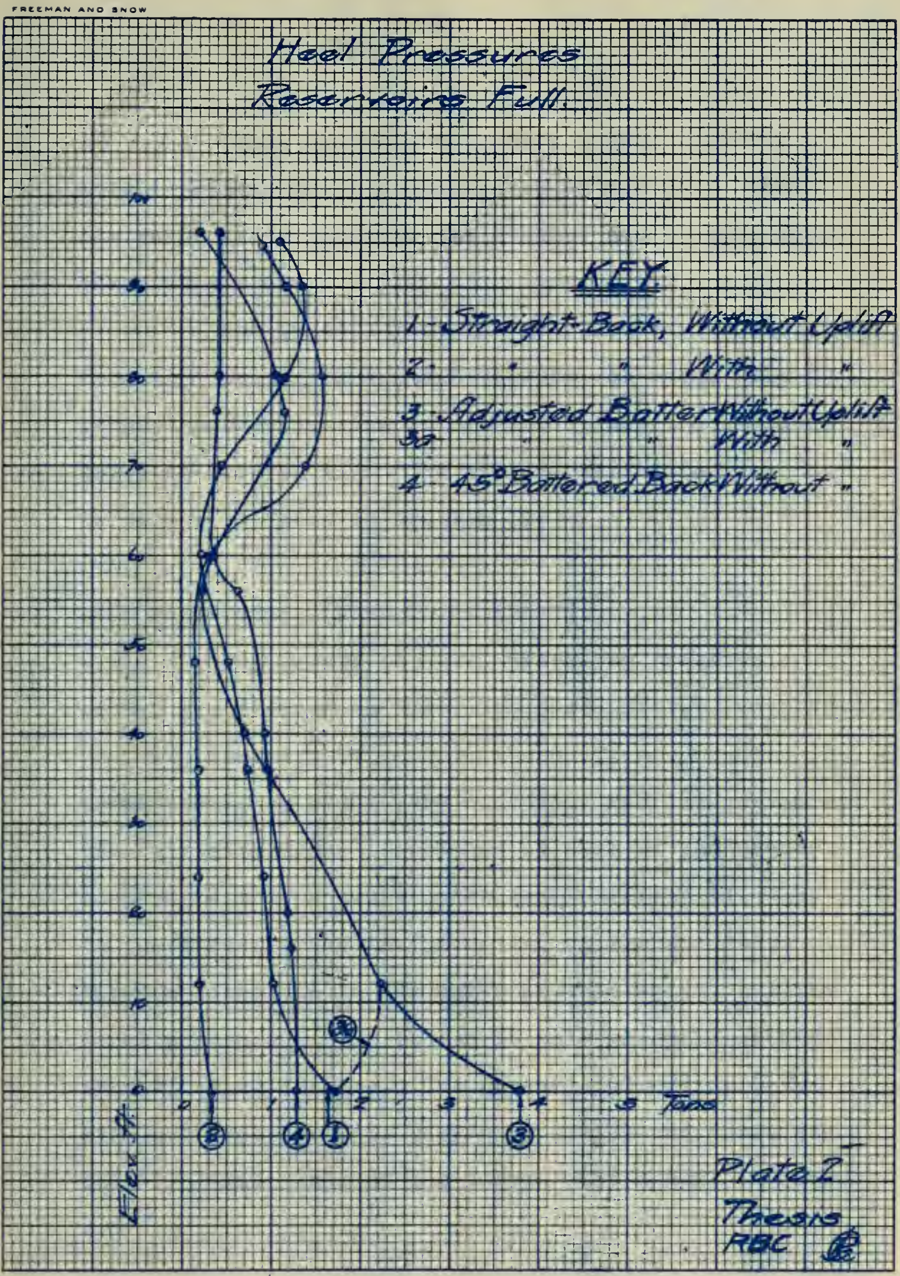
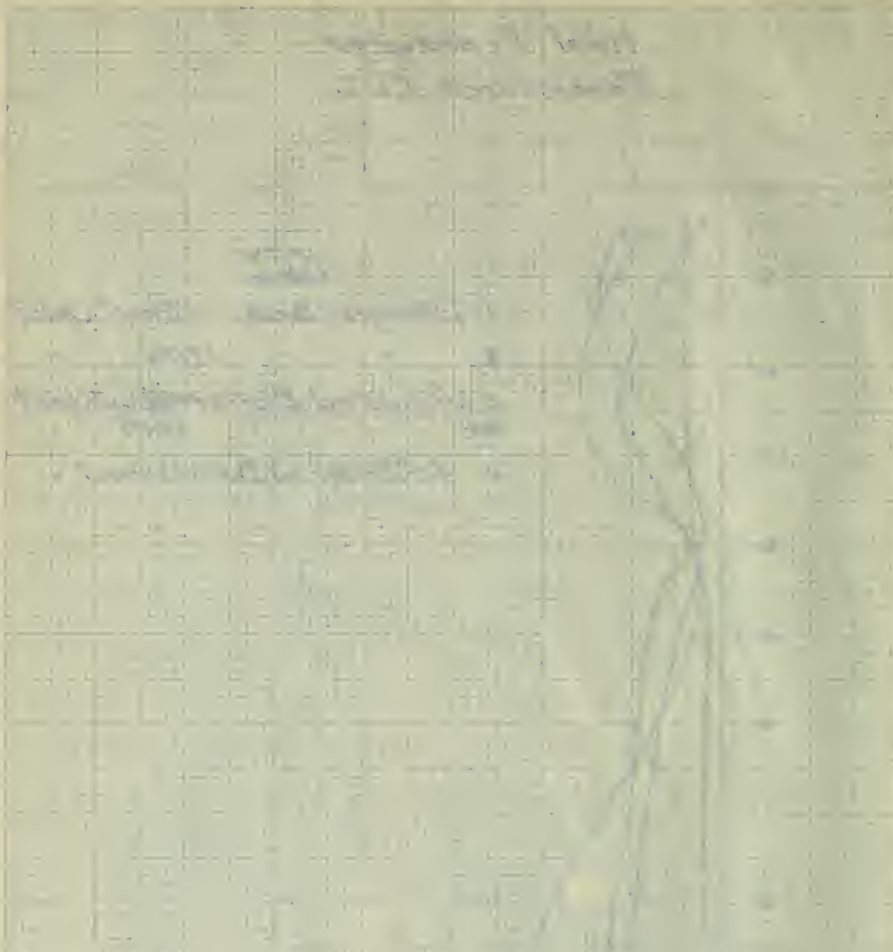

FREEMAN AND SNOW
Heel Pressures
Reservoirs Full.
KEY
1 - Straight Back, Without Uplift
2 - " " With "
3 - Adjusted Batter Without Uplift
3a " " With "
4 - 45° Battered Back Without "
100
90
80
70
60
50
40
30
20
10
0
0
1
2
3
4
5 Tons
Elev. ft.
2
4
1
3
3a
Plate I
Thesis
RBC



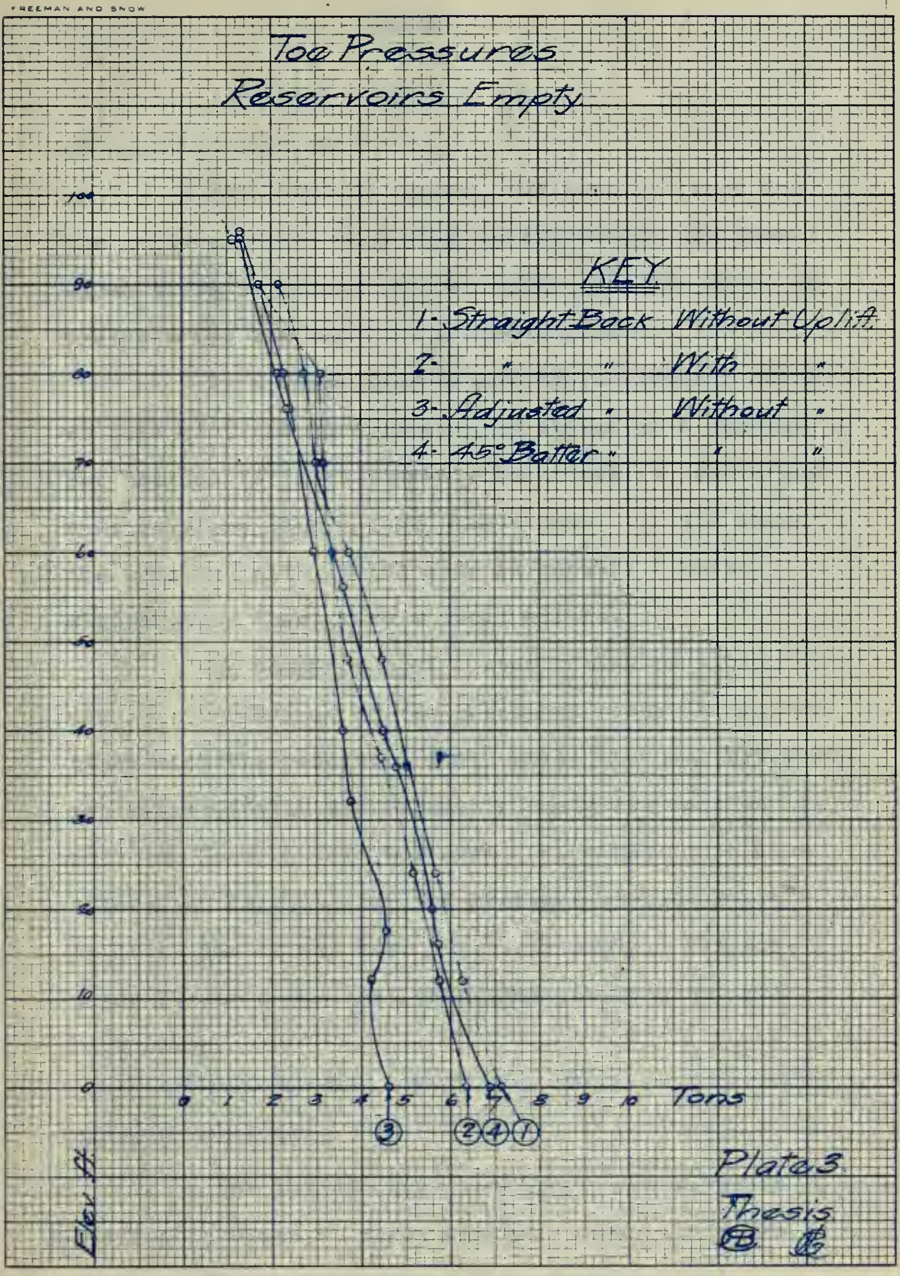
These curves show a very marked similarity at all points, with the exception of curve 3. This one varies a little from the others, due to particular shape of the dam.

The intensity of the pressure varies uniformly from the top of the section towards the base, the largest pressure being in Section I, and the smallest being on Section III. The maximum variation from top to bottom is found in Section IV.

HEEL PRESSURES: (RESERVOIRS EMPTY).

The curves for heel pressures, reservoir empty, are plotted on Plate IV, Page XXXII. Curve No.4.

Section IV, due to the uniform variation in cross sections, has a very uniformly varying pressure, the only difference being that at each of the denser diaphragms the pressure increases. Section III, due to the sudden break in the batter at the 40' line, shows a large dip in the curve at this point, but the final pressure at the bottom is only slightly greater than it is at the top. The pressure lines for both straight back sections vary very little and cross at the 60' line. Both of the curves show a smaller stress


FREEMAN AND SNOW
Toe Pressures
Reservoirs Empty
KEY
1- Straight Back Without Uplift
2- " " With "
3- Adjusted " Without "
4- 45° Batter " " "
100
90
80
70
60
50
40
30
20
10
0
0 1 2 3 4 5 6 7 8 9 10 Tons
3
2
4
1
Elev. ft.
Plate 3
Thesis

at the base than at the top.

Both sets of curves, the straight back and the battered back, show that uplift causes the pressures to be in general concave away from the lines of pressure for the sections without uplift.

By a comparison of curves on Plates I and II we find that the distribution of stress is a minumum for Section III, considered without uplift, while the same section, with uplift, comes next. In all cases the sections with uplift show in general a better distribution than those without uplift.

By comparison of curves on Tables I, II, III and IV, the maximum of toe pressures occur in all sections with the reservoir full, while the heel pressures are greatest with the reservoir empty for Section II, full for Section III, but again, when uplift is considered in Section III, the pressure at heel falls to less than it was with reservoir empty. For Sections IV and I the heel pressures are greatest with high water..

FACTORS OF SAFETY:

For the straight back type, the curve of Section I, Plate V, shows abundant strength in the upper portion, with a minumum at the 50' level,

at the base than at the top.

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For the straight back type, the curve of Section I, Plate V, shows abundant strength in the upper portion, with a minimum at the 50' level,

increasing uniformly thereafter to a maximum of 2.5 at the base.

Section II indicates a critical point at the same elevation as the first, but for the proportions of this design the factor (1.8) is considerably less than desirable in practice. The minimum should be at least 2.0.

Though the section is stable, so far as compliance with general principles is concerned, the factor of safety developed is less than corresponding points of Section I in all cases, as is to be expected, for the reason, that uplift so largely increases the overturning moment.

The battered back type of Plates X and XI requires a larger factor of safety, since the material of the interior is of losse construction, and no dependence is placed upon its shearing strength.

Even so, the factors are far in excess of the required minimum, as a glance at Plate Va will verify. The mimimum occurs, for uplift, at the base of the adjusted back section, as was explained in the detailed discussion of the design of that particular section.

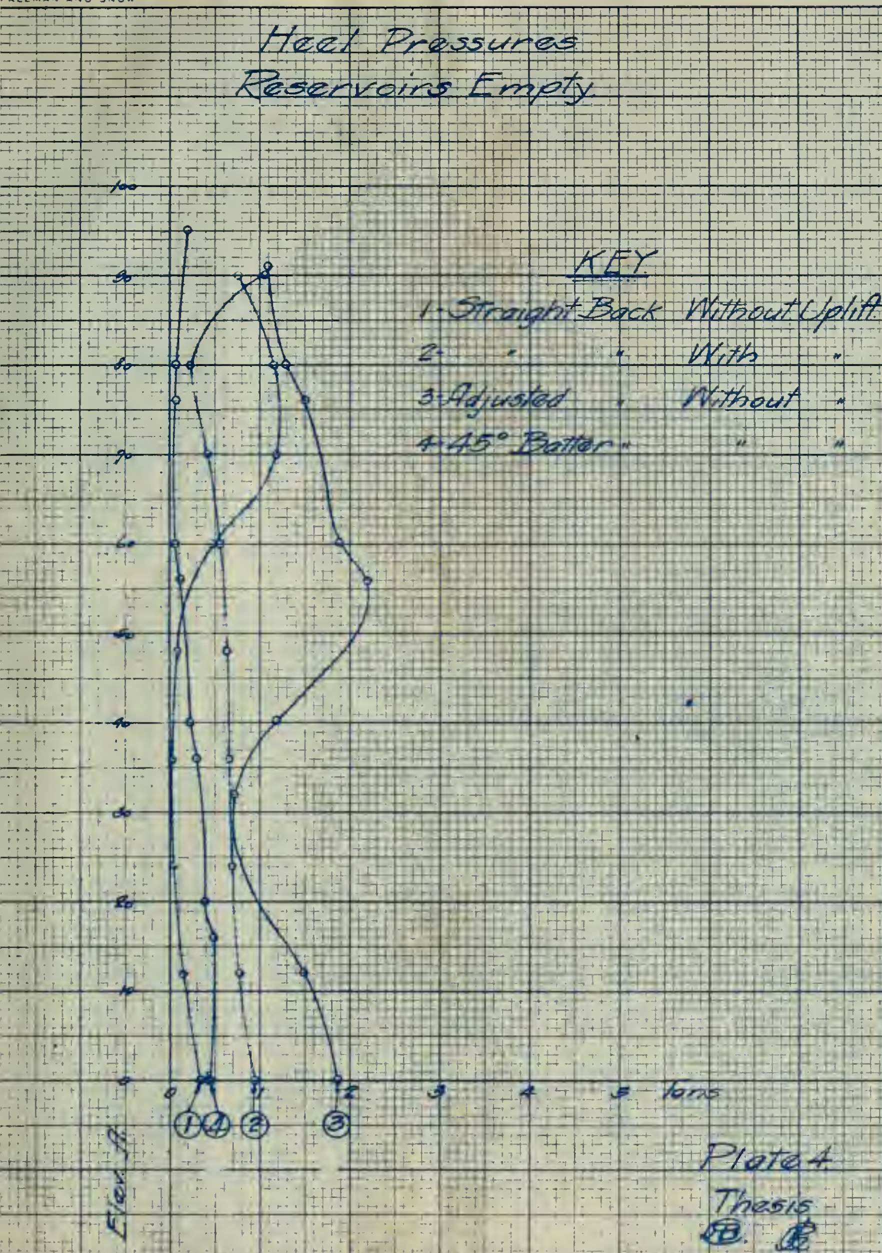
increasing uniformly thereafter to a maximum of 2.5 at the base.

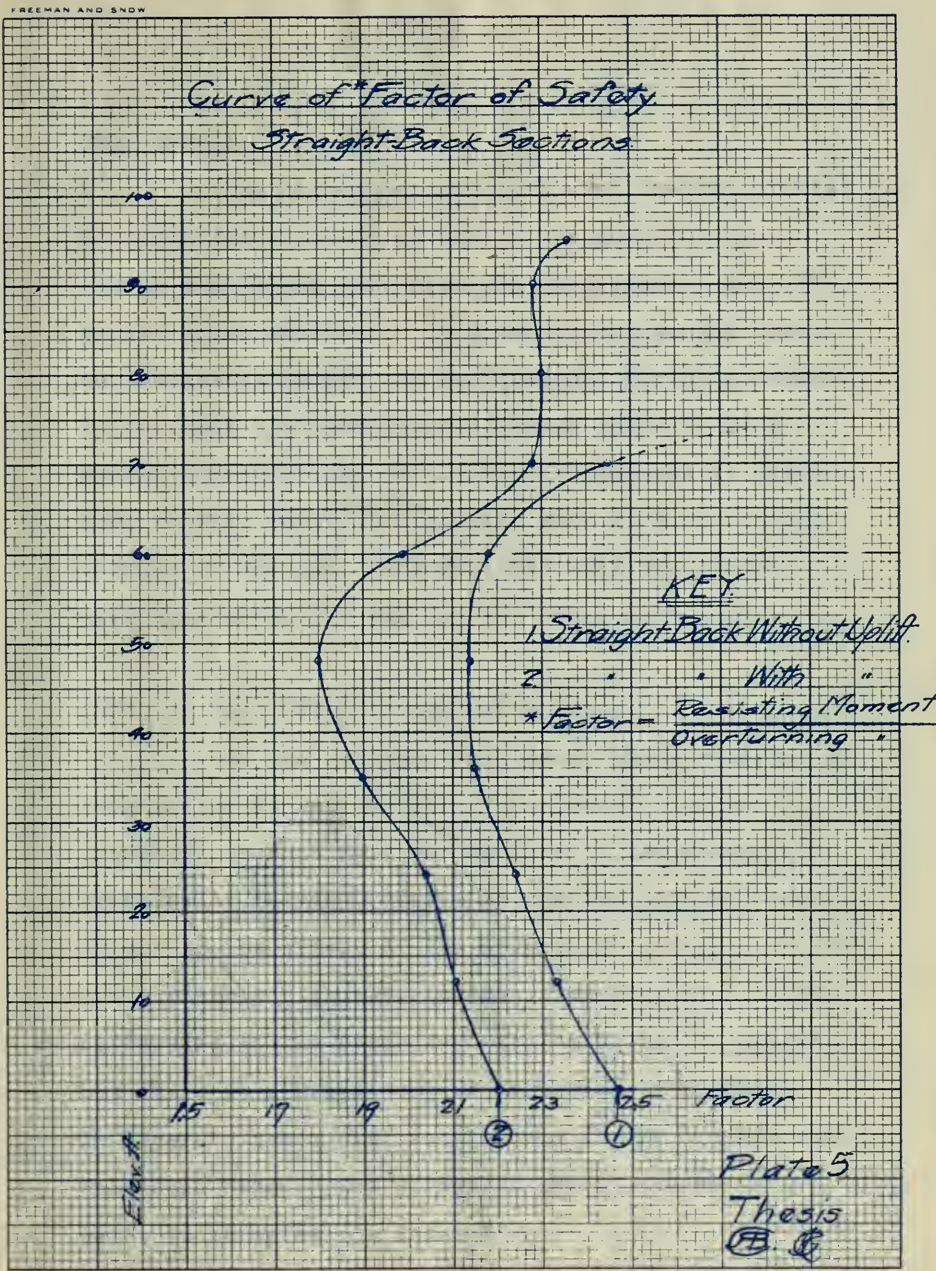
Section II indicates a critical point at the same elevation as the first, but for the proportions of this design the factor (1.8) is considerably less than desirable in practice. The minimum should be at least 2.0.

Though the section is stable, so far as compliance with general principles is concerned, the factor of safety developed is less than corresponding points of Section I in all cases, as is to be expected, for the reason, that uplift so largely increases the overturning moment.

The battered back type of Plates X and XI requires a larger factor of safety, since the material of the interior is of loose construction, and no dependence is placed upon its shearing strength.

Even so, the factors are far in excess of the required minimum, as a glance at Plate Va will verify. The minimum occurs, for uplift, at the base of the adjusted back section, as was explained in the detailed discussion of the design of that particular section.


Heel Pressures
Reservoirs Empty
KEY
1-Straight Back Without Uplift
2- " " With "
3-Adjusted " Without "
4-45° Batter " " "
100
90
80
70
60
50
40
30
20
10
0
0 1 2 3 4 5 Tons
1 4 2 3
Elev. ft.
Plate 4
Thesis
B. B.

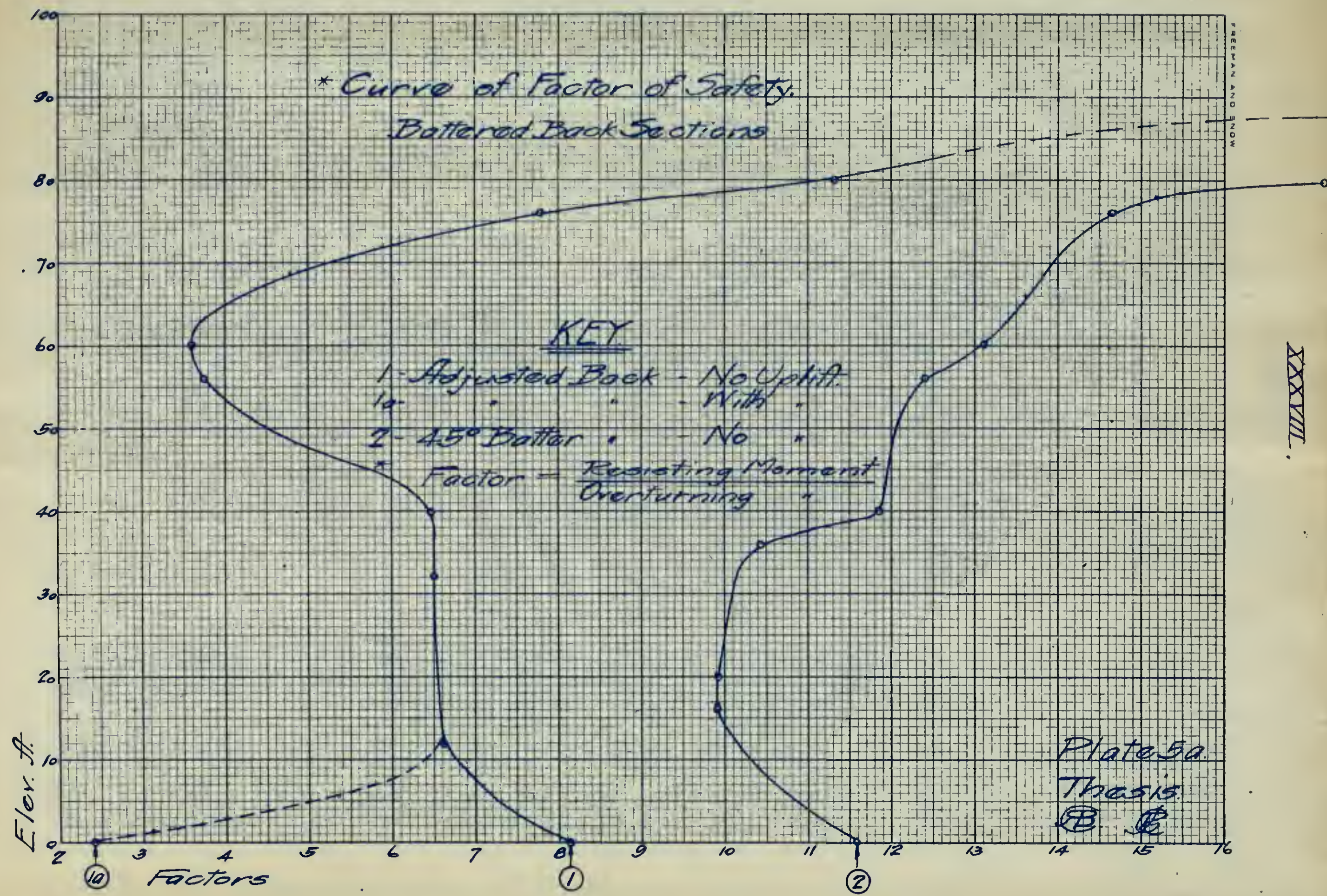

FREEMAN AND SNOW
Curve of *Factor of Safety
Straight-Back Sections
100
90
80
70
60
50
40
30
20
10
0
1.5
1.7
1.9
2.1
2.3
2.5
Factor
2
1
Elev. ft.
KEY
1. Straight Back Without Uplift.
2. " " With "
* Factor = Resisting Moment / Overturning "
Plate 5
Thesis

UNIT SHEARS:

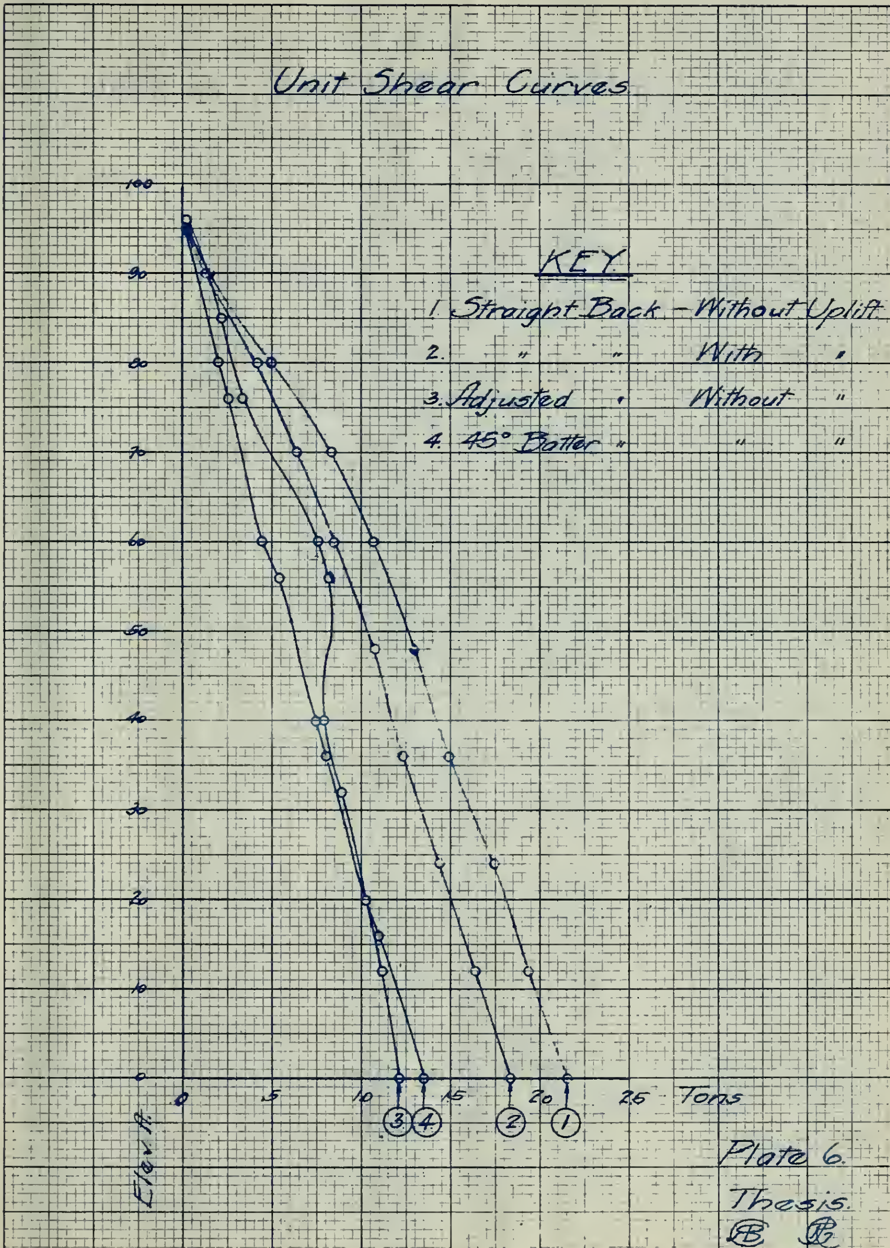
The unit shears for all sections are plotted without regard to the nature of the material, as though the stress were distributed uniformly over the entire cross section. On this assumption the shear varies uniformly for all the sections having a uniform batter, but with Section III, where there is such a sudden change at the 56' line, the increase in shear down to that point changes more rapidly than it does for the balance of the dam. The battered back dams show a much smaller unit shear for the other sections.

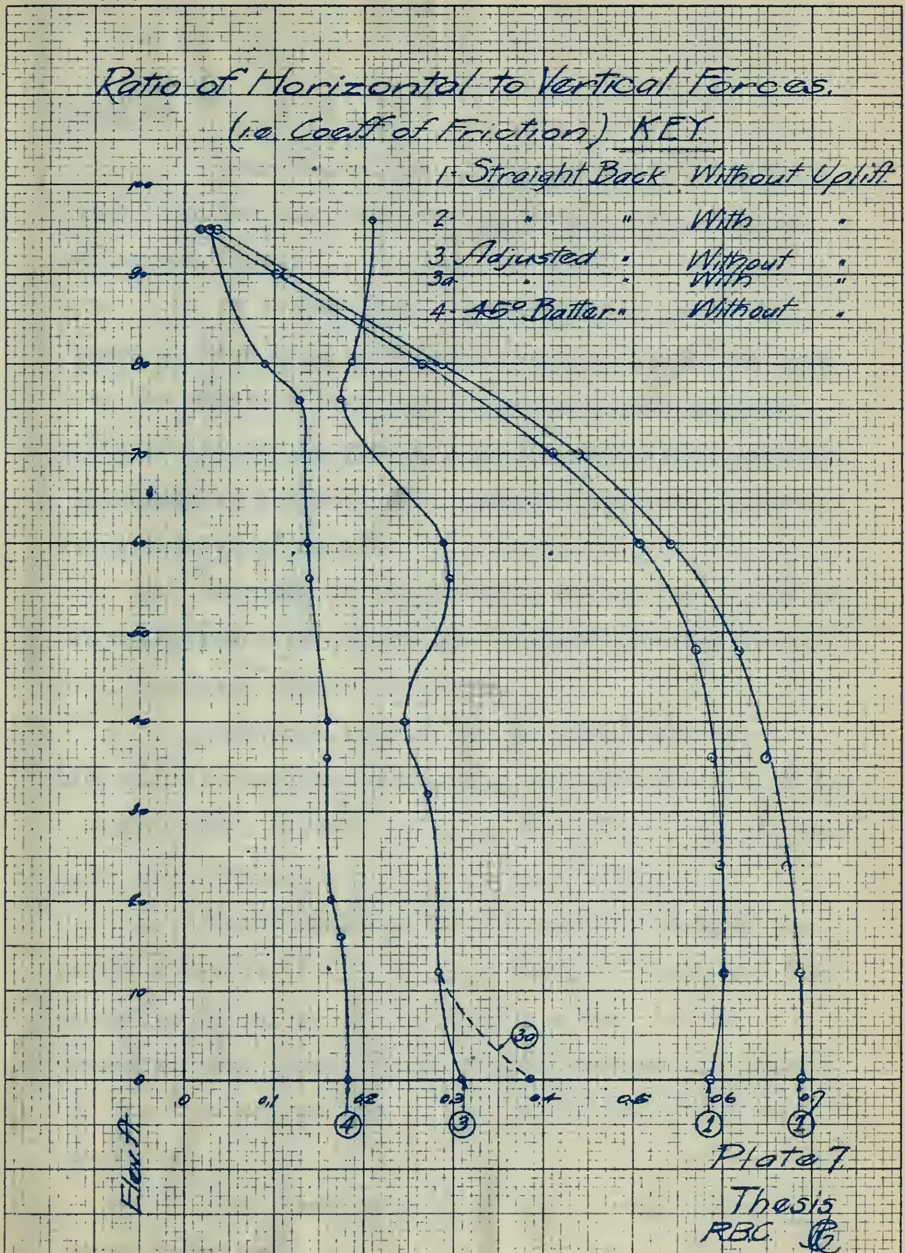
RATIO OF HORIZONTAL TO VERICAL FORCES:

In the straight back dams, the variation in this factor is very uniform, the rate of change gradually decreasing. For Section IV the change is practically nothing, except at the top. This change is due to the effect of the weight of that part of the section above high water mark. For Section III the difference in the ratio from top to bottom is slight, but there is a great deal of variation in between. In general, the curves for III and IV are concave away from each other. The effect of uplift causes the ratio to go up at the foundation.



* Curve of Factor of Safety.
Battered Back Sections
KEY
1 - Adjusted Back - No Uplift.
1a - " " - With "
2 - 45° Batter " - No "
* Factor = Resisting Moment / Overturning "
Elev. ft.
Factors
100
90
80
70
60
50
40
30
20
10
0
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
1a
1
2
Plate 5a
Thesis
FREEMAN AND SNOW


Unit Shear Curves
KEY
1. Straight Back - Without Uplift
2. " " With "
3. Adjusted " Without "
4. 45° Batter " " "
100
90
80
70
60
50
40
30
20
10
0
0
5
10
15
20
25
Tons
Elev. ft.
3
4
2
1
Plate 6.
Thesis.


Ratio of Horizontal to Vertical Forces.
(i.e. Coeff. of Friction) KEY.
1- Straight Back Without Uplift
2 " " With "
3 Adjusted " Without "
3a " " With "
4- 45° Batter " Without "
Elev. ft.
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
4
3
1
1
3a
Plate 7.
Thesis
RBC.

SUMMARY:

From the points of this investigation which stand out the most prominently, the following conclusions are drawn:

1. It is impossible to contruct a dam of the battered back type of as small sectional area as that for the straight back type, for the reason that the pressure due to an increase in the quantity of masonry, increases at a slower rate than the attendant increase in the force of uplift.

2. Economy in construction may be had by the use of a cheaper material for the bulk of interior which will transmit direct pressures..

3. The stability of the battered back type against sliding and overturning far exceeds that of the vertical-back section, and the distribution of stresses over any horizontal section is more uniform..

4. Under-drainage, while unpractical with the solid straight back type, is a natural outcome of the peculiar nature of the construction used in the battered back type. This drainage reduces the uplift in the body of the dam, to a point where it may be neglected.

This construction, however, calls for a water proof back.

SUMMARY:

From the points of this investigation which stand out the most prominently, the following conclusions are drawn:

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This construction, however, calls for a water proof face.

It is useless to draw a comparison as to the relative economy of the two types, without going into the details of cost for some particular case. It would seem that wherever the stone for use in the battered back type could be had from quarries convenient to the site, that a great saving in cost could be made over a concrete dam of the vertical back type. Much depends also on whether the cost of excavation for the larger foundation would offset the saving made in cheaper construction of the body of the dam.

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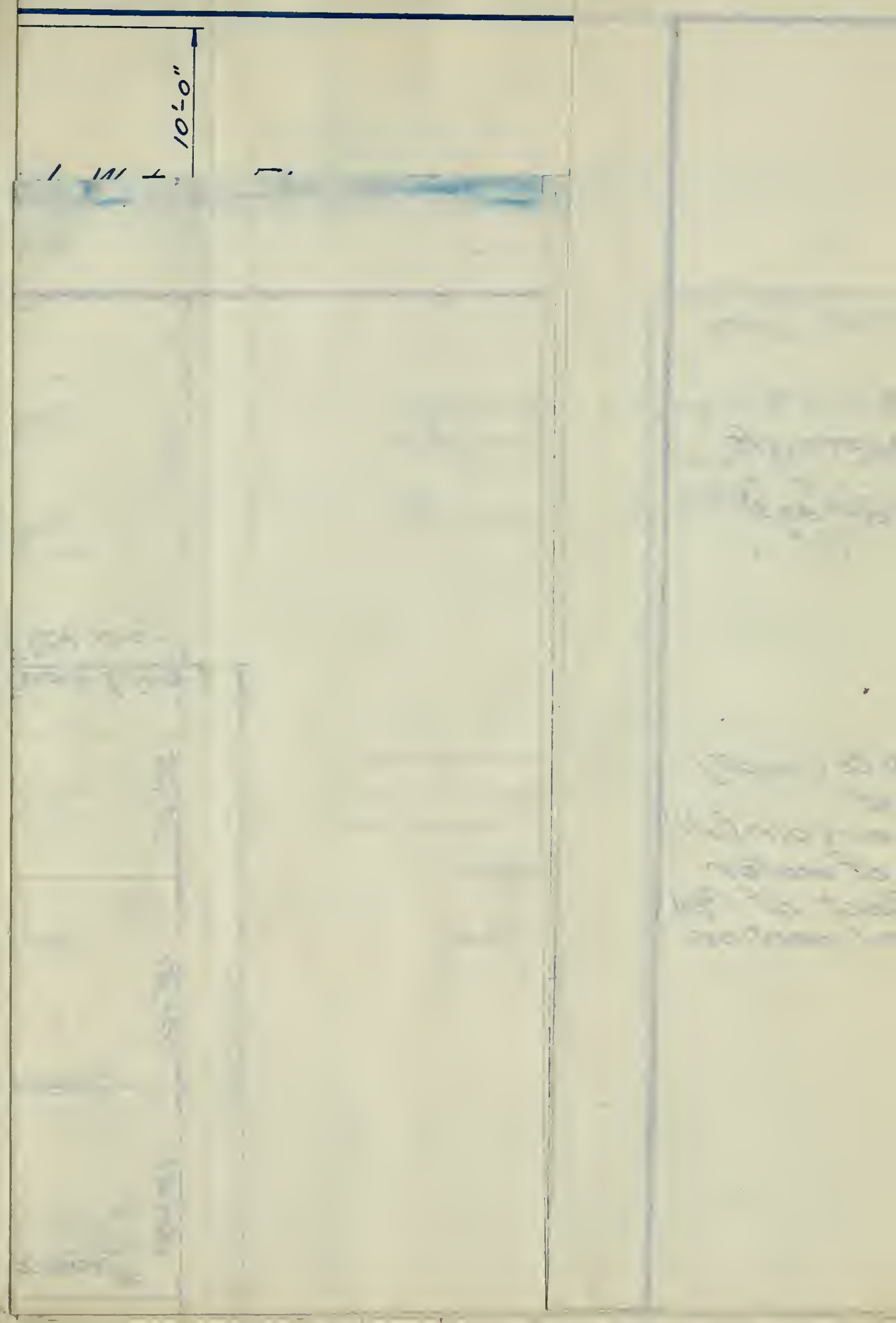
10'-0"

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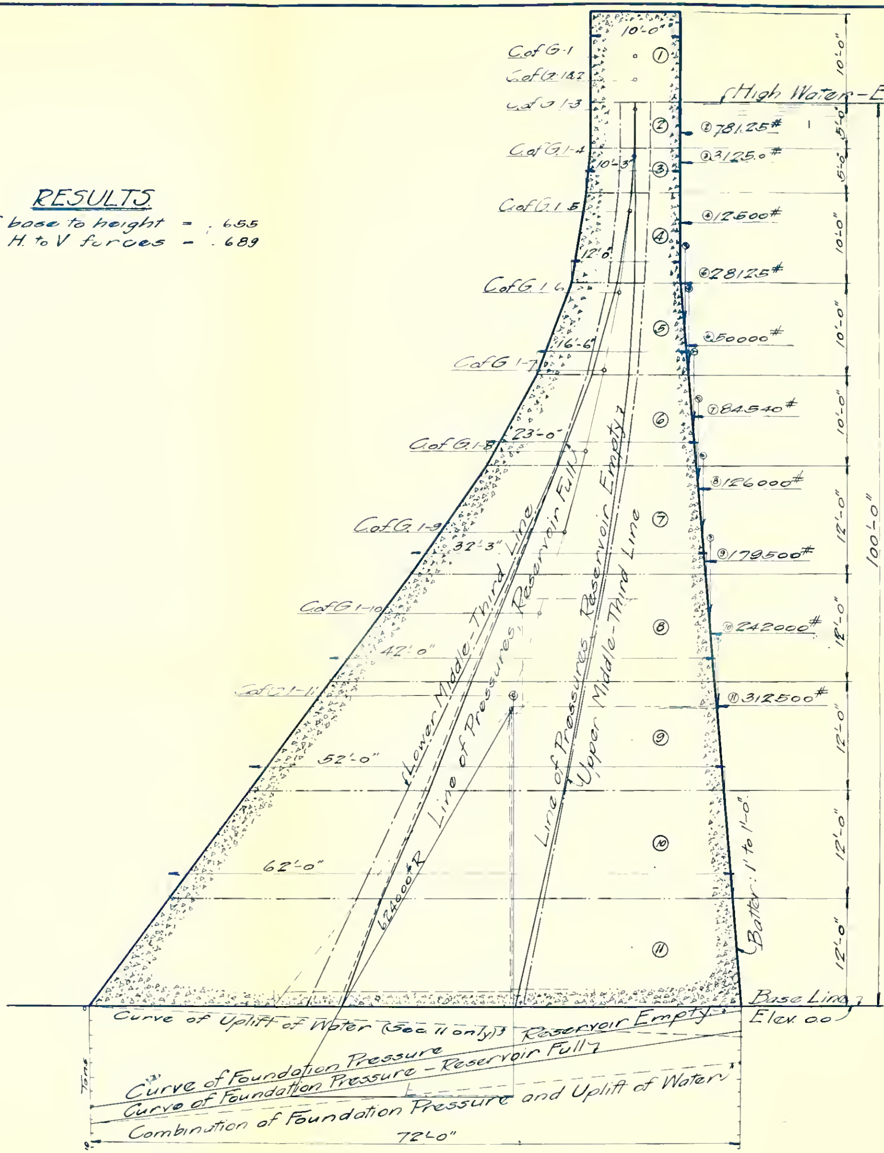
RESULTS

Ratio of base to height = .655
" " H. to V. forces = .689



SECTION I.

DATA

Head of water	100 ft
Weight of "	62.5 lb per cu ft
" " concrete	150 " " " "
Working pressure, toe	14 tons " sq ft
" " heel	18 " " " "

KEY

Pressure Lines - Full & Empty
Limits of Middle-Third
Pressure Line (reservoir full) neglecting vertical pressure of water.
Construction for effect of (2/3) uplift considered only for total section

Vertical Components		
Section		Pressure
5	5	108.0 #
5 & 6	6	261.0
5 to 7	7	514.0
5 to 8	8	802.0
5 to 9	9	1167.0
5 to 10	10	1592.0
5 to 11	11	2080.0

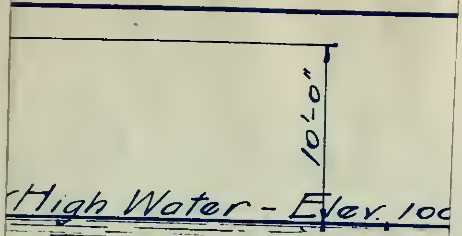
THESIS.

STRAIGHT-BACK CONCRETE DAM
CONSIDERED WITHOUT UPLIFT.

Plate 8. Scale 1/8" = 1'-0"

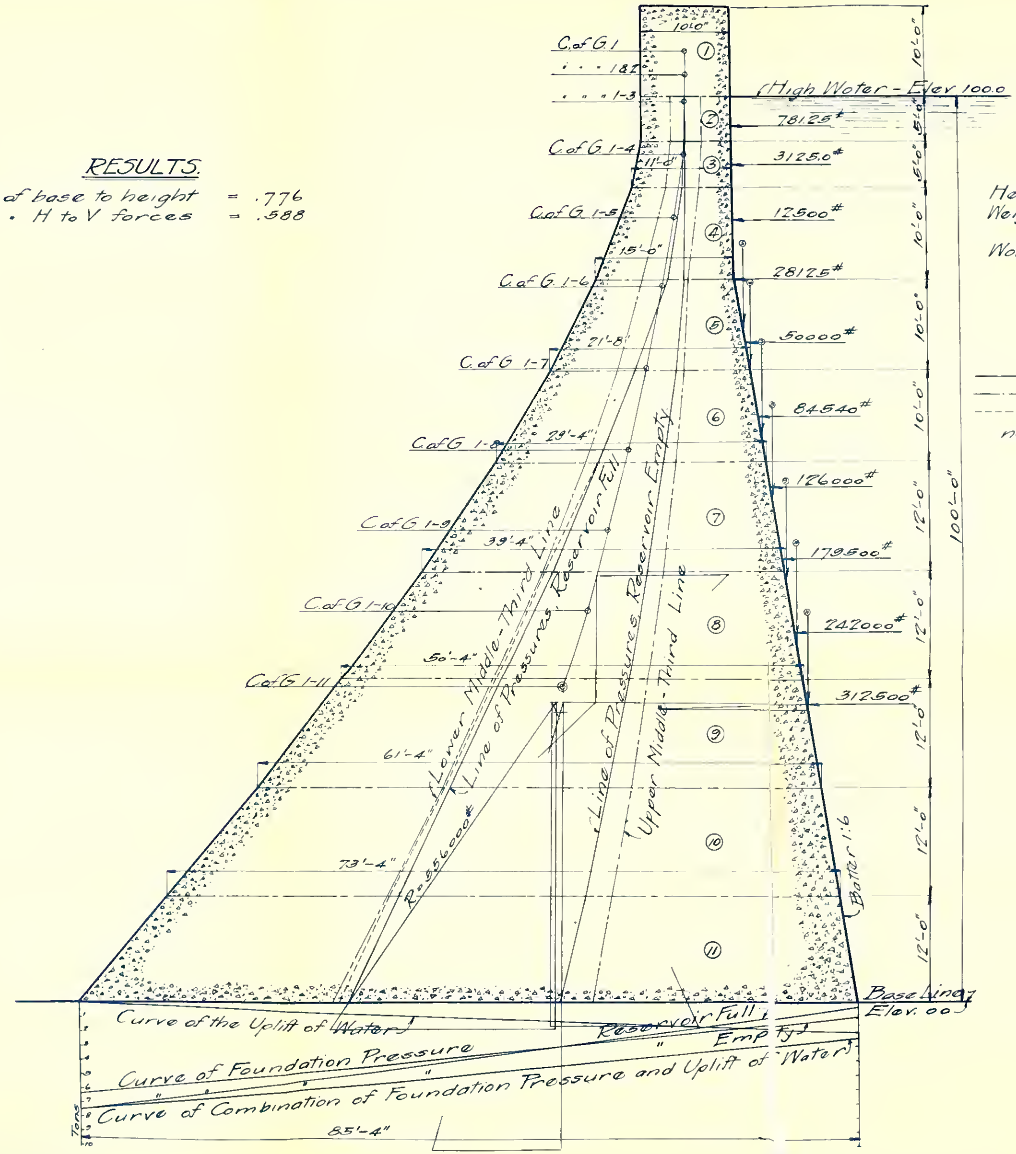
May 1912.

R. Geisler Ronald B. Clark


10'-0"
High Water - Elev. 10

RESULTS.

Ratio of base to height = .776
" " H to V forces = .588



SECTION II.

DATA.

Head of water	100 ft
Weight " "	62.5 lbs per cu ft
" " concrete	150 " " " "
Working pressure, toe:	14 tons " sq "
" " heel	18 " " " "

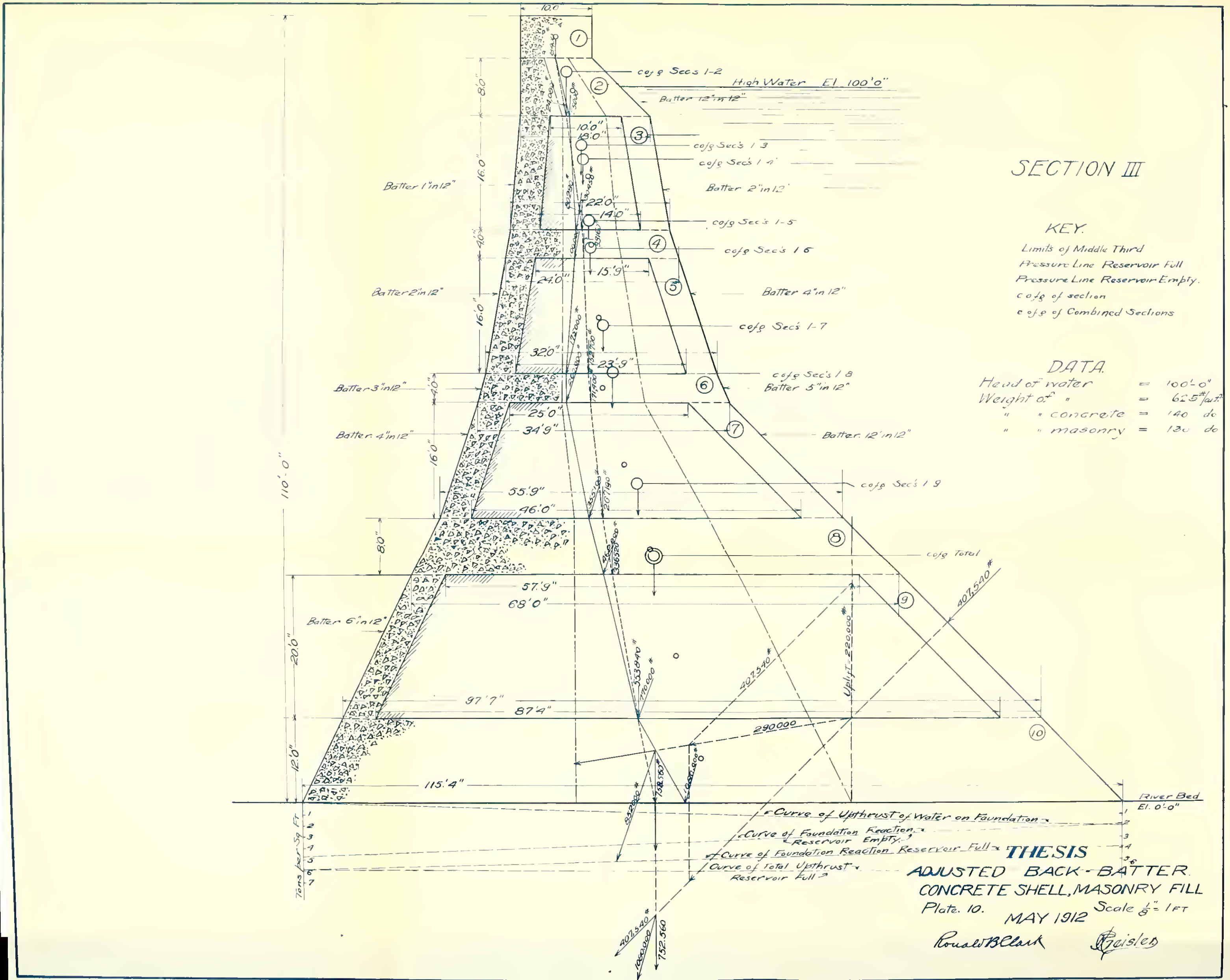
KEY

———— Pressure Lines – Full & Empty.
—·—·— Limits of Middle-Third
-------- Pressure Line (reservoir full) neglecting vertical pressure of water

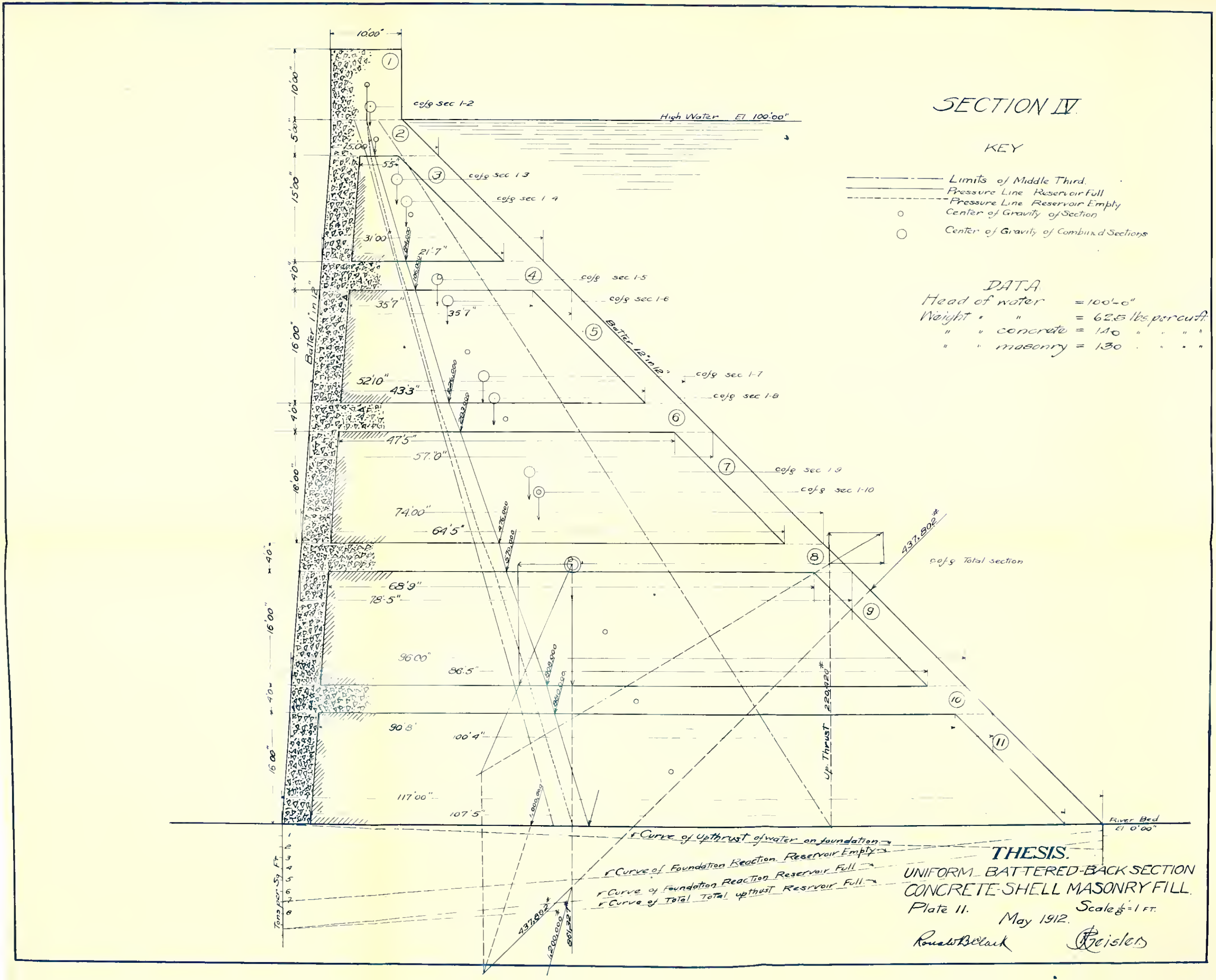
Vertical Components		
Section	Mk	Pressure
5	5	434 lbs.
5&6	6	1022
5 to 7	7	2000
5 - 8	8	3207
5 - 9	9	4665
5 - 10	10	6373
5 - 11	11	8331

THESIS.
STRAIGHT-BACK CONCRETE DAM.
DESIGNED FOR UPLIFT.
Plate 9. Scale $\frac{1}{8}$"=1'-0"
May 1912.
Geisler
Ronald B. Clark

h Water El. 100'0"



SECTION III
KEY.
Limits of Middle Third
Pressure Line Reservoir Full
Pressure Line Reservoir Empty.
c of g of section
c of g of Combined Sections
DATA.
Head of water = 100'-0"
Weight of " = 62.5#/cu.ft.
" " concrete = 140 do
" " masonry = 130 do
High Water El 100'0"
River Bed El. 0'-0"
Batter 1"in 12"
Batter 2"in 12"
Batter 3"in 12"
Batter 4"in 12"
Batter 6"in 12"
Batter 12"in 12"
Batter 5"in 12"
c of g Secs 1-2
c of g Sec's 1 3
c of g Sec's 1 4
c of g Sec's 1-5
c of g Sec's 1 6
c of g Sec's 1-7
c of g Sec's 1 8
c of g Sec's 1 9
c of g Total
110'0"
115'4"
97'7"
87'4"
68'0"
57'9"
55'9"
46'0"
34'9"
25'0"
32'0"
23'9"
24'0"
15'9"
22'0"
14'0"
10'0"
18'0"
10.0'
407,540#
290000
Uplift 220000#
752,560
Curve of Upthrust of Water on Foundation
Curve of Foundation Reaction Reservoir Empty
Curve of Foundation Reaction Reservoir Full
Curve of Total Upthrust Reservoir Full
Tons per Sq Ft
THESIS
ADJUSTED BACK-BATTER.
CONCRETE SHELL, MASONRY FILL
Plate. 10.
MAY 1912
Scale 1/8" = 1 FT
Ronald B Clark
Geisler



SECTION IV.
KEY
Limits of Middle Third.
Pressure Line Reservoir Full
Pressure Line Reservoir Empty
Center of Gravity of Section
Center of Gravity of Combined Sections
DATA.
Head of water = 100'-0"
Weight " " = 62.5 lbs per cu ft.
" " concrete = 140 " " "
" " masonry = 130 " " "
High Water El 100'00"
Batter 1' in 12"
Batter 12" in 12"
c of g Total section
Up Thrust 220,420#
437,802#
River Bed El 0'00"
Curve of upthrust of water on foundation
Curve of Foundation Reaction, Reservoir Empty
Curve of Foundation Reaction, Reservoir Full
Curve of Total upthrust Reservoir Full
Tons per Sq Ft
THESIS.
UNIFORM BATTERED-BACK SECTION
CONCRETE-SHELL MASONRY FILL.
Plate 11.
May 1912.
Scale 1/8" = 1 FT.